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CASE REPORT

Optimal estimators in misspecified linear regression model with an application to real-world data

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ABSTRACT
In this article, we propose the sample information optimal estimator and the stochastic restricted optimal estimator for misspecified linear regression model when multicollinearity exists among explanatory variables. Further, we obtain the superiority conditions of proposed estimators over some other existing estimators in the mean square error matrix criterion in a standard form which can apply to all estimators considered in this study. Finally, a real-world example and a Monte Carlo simulation study are presented for the proposed estimators to illustrate the theoretical results.

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Sample information optimal estimator; stochastic restricted optimal estimator; mean square error matrix

1. Introduction

The multiple linear regression model defined as

$$y = X\beta + \epsilon, \tag{1}$$

where y is the $n \times 1$ vector of observations on the predictor variable, X is the $n \times p$ matrix of observations on p nonstochastic regressor variables, β is a $p \times 1$ vectors of unknown parameters, ϵ is the $n \times 1$ vector of disturbances, such that $E(\epsilon) = 0$ and $E(\epsilon\epsilon') = \Omega = \sigma^2 I$.

The estimator for β considered commonly in practical situations is the ordinary least-squares estimator (OLSE)

$$\hat{\beta}_{OLSE} = (X'X)^{-1}X'y, \tag{2}$$

which is unbiased and has the minimum variance among all linear unbiased estimators.

If the columns of the X matrix are nearly linearly dependent, i.e., multicollinear, then the matrix $X'X$ is almost singular. Consequently, the numerical computation of Eq. (2) will be unstable, and the variance of the OLSE will be large. As a remedial measure to the multicollinearity problem, biased estimators have been used in the literature. Some of the biased estimators are based only on model Eq. (1), namely ridge estimator (RE) (Hoerl and Kennard 1970), almost unbiased ridge estimator (AURE) (Singh, Chaubey, and Dwivedi 1986), Liu estimator (LE) (Liu 1993), almost unbiased Liu estimator (AULE) (Akdeniz and Kairanlar 1995), principal component regression estimator (PCRE) (Massy 1965), r-k class estimator (Baye and Parker 1984) and r-d class estimator (Kaçiranlar and Sakalloğlu 2001), and are given as

$$\hat{\beta}_{RE} = (X'X + kI)^{-1}X'X\hat{\beta}_{OLSE}, \tag{3}$$

$$\hat{\beta}_{AURE} = \left(I - k^2(X'X + kI)^{-2}\right)\hat{\beta}_{OLSE}, \tag{4}$$

$$\hat{\beta}_{LE} = (X'X + I)^{-1}(X'X + dI)\hat{\beta}_{OLSE}, \tag{5}$$

$$\hat{\beta}_{AULE} = \left(I - (1-d)^2(X'X + I)^{-2}\right)\hat{\beta}_{OLSE}, \tag{6}$$

$$\hat{\beta}_{PCRE} = T_h T_h'\hat{\beta}_{OLSE}, \tag{7}$$

$$\hat{\beta}_{rk} = T_h T_h'(X'X + kI)^{-1}X'X\hat{\beta}_{OLSE}, \tag{8}$$

$$\hat{\beta}_{rd} = T_h T_h'(X'X + I)^{-1}(X'X + dI)\hat{\beta}_{OLSE}, \tag{9}$$

respectively, where $k > 0$ and $0 < d < 1$ are the shrinkage parameters, and $T_h = (t_1, t_2, ..., t_h)$ is the first h columns of the standardized eigenvectors of $X'X$ represents by $T = (t_1, t_2, ..., t_h, ..., t_m)$.

According to the literature, some other biased estimators are also available based on model Eq. (1) and prior information about β in the form of exact linear restrictions or stochastic linear restrictions. Theil

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